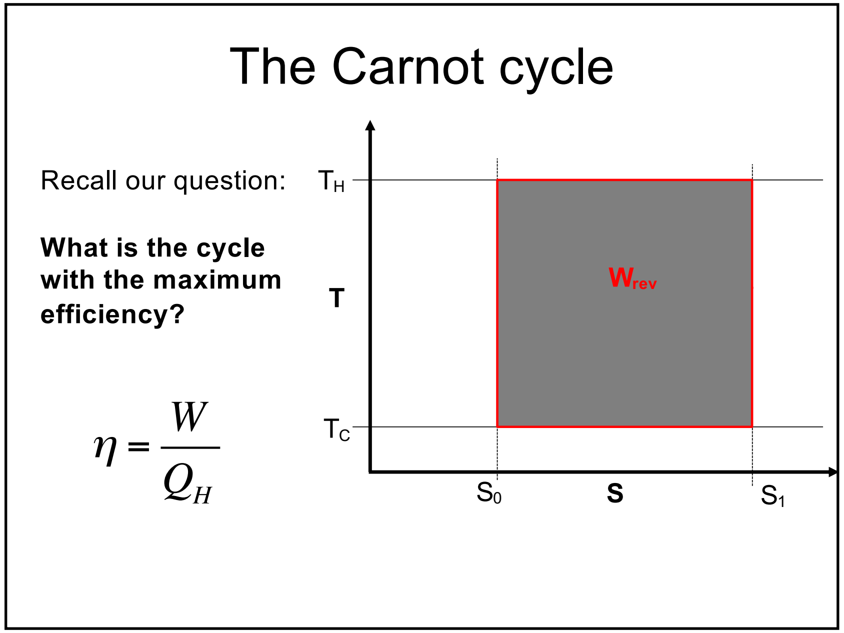
Sustainable Energy Systems

1. Technical aspects of energy

Outline of Part 1: Thermo

Objective: Derive and understand the physical laws that characterize and limit energy conversion systems

- 1st Law of Thermodynamics
  - Internal energy
  - Work
  - Enthalpy
- 2nd Law of Thermodynamics
  - Entropy
  - Reversible and irreversible processes
  - State functions
- Heat to work conversion
  - T-S diagrams
  - **Idealized systems (Carnot cycle)**
- Real heat to work and work to heat conversion systems
  - Rankine cycles
  - Refrigeration cycles and heat pumps
  - Engines
- Exergy: calculating the maximum work that can be produced/recovered
- Electrical systems
  - Electrical machines
  - Fuel cells

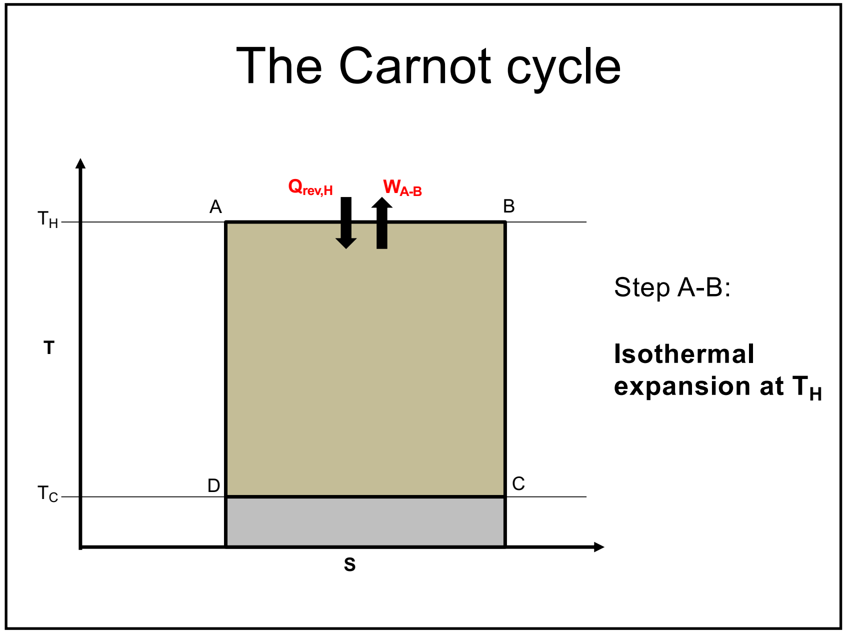


The Carnot cycle

Recall our question:

What is the cycle with the maximum efficiency?

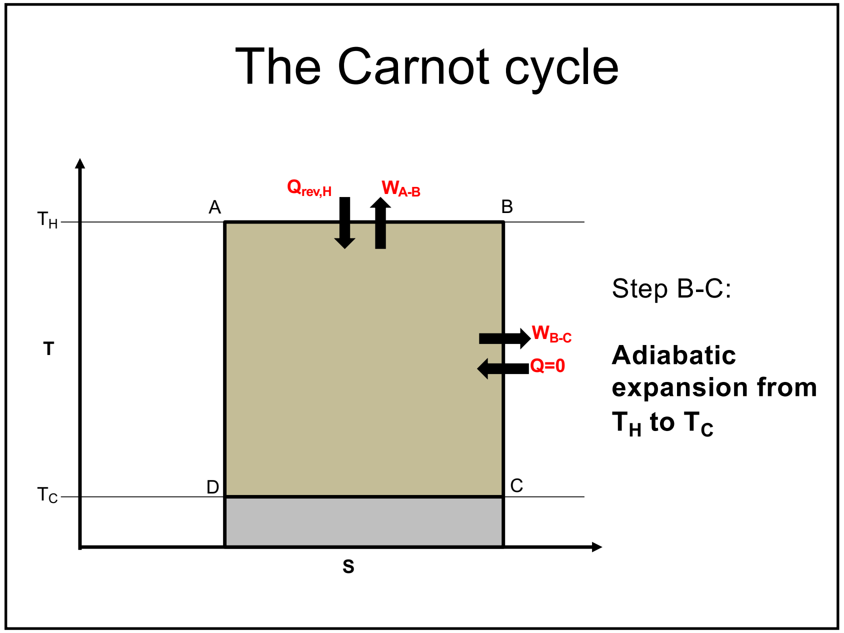
$$\eta = \frac{W}{Q_H}$$



The Carnot cycle

Step A-B:

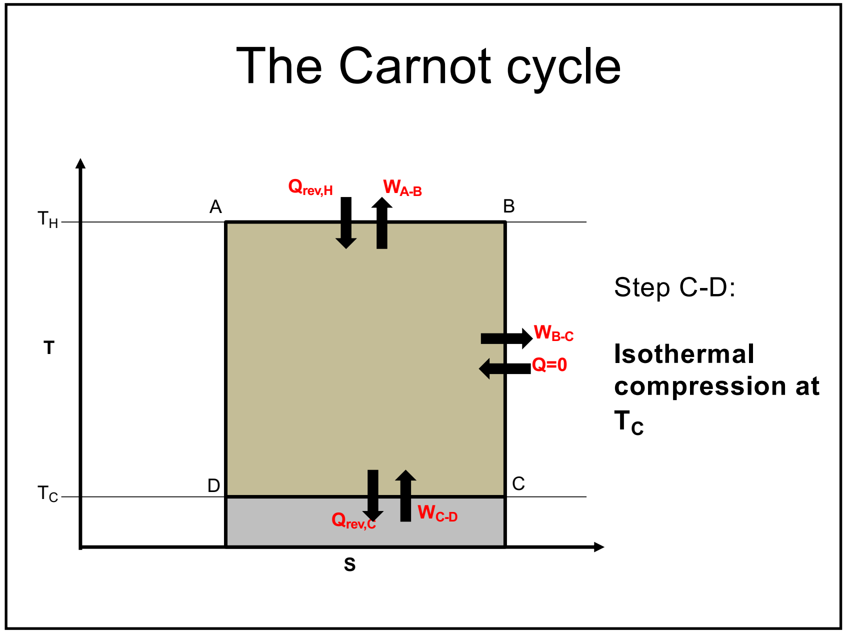
Isothermal expansion at T_H



The Carnot cycle

Step B-C:

Adiabatic expansion from T_H to T_C

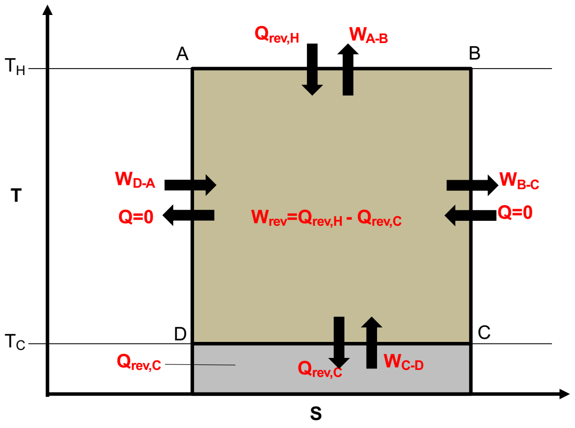


The Carnot cycle

Step C-D:

Isothermal compression at T_C

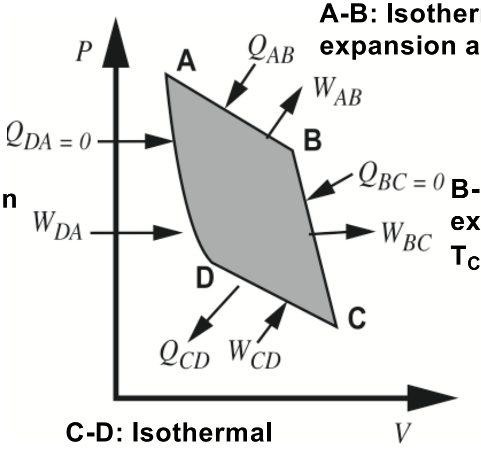
The Carnot cycle



Step DA:

Adiabatic compression to T_H

The Carnot cycle



A-B: Isothermal expansion at T_H

D-A: Adiabatic compression to T_H

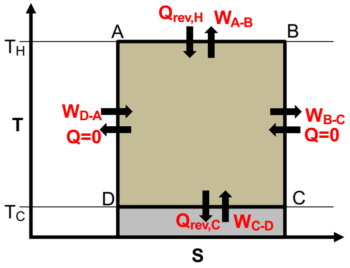
B-C: Adiabatic expansion to T_C

C-D: Isothermal compression at T_C

The Carnot cycle

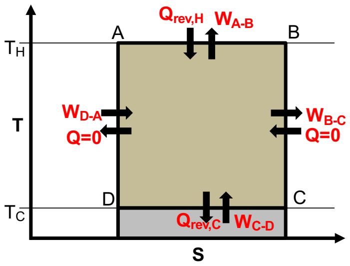
What is the maximum efficiency?

$$\eta = \frac{W}{Q_H}$$



$$\eta = \frac{W}{Q_H} = \frac{-W_{tot}}{Q_{AB}} = \frac{-(W_{AB} + W_{BC} + W_{CD} + W_{DA})}{Q_{AB}}$$

The Carnot cycle



Step A-B: Isothermal expansion

T= cst U=f(T)≠f(P,V) for ideal gases:

$$\Delta U = Q + W = 0 \;\rightarrow\; Q_{AB} = -W_{AB}$$

$$= \int p\,dV = RT_H \int \frac{dV}{V} =$$

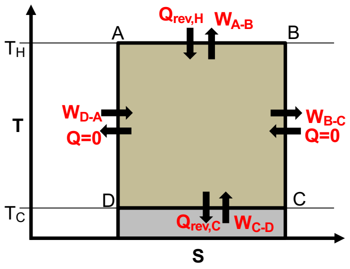
$$RT_H \ln\frac{V_B}{V_A} = RT_H \ln\frac{P_A}{P_B}$$

Step A-B

$$Q_{AB} = -W_{AB} =$$

$$RT_H \ln\frac{P_A}{P_B}$$

The Carnot cycle



Step B-C: Adiabatic expansion

U=f(T) and Cv=cst (for ideal gases):

$$\Delta U = Q + W = W_{BC} =$$
$$C_V \Delta T = C_V (T_C - T_H)$$

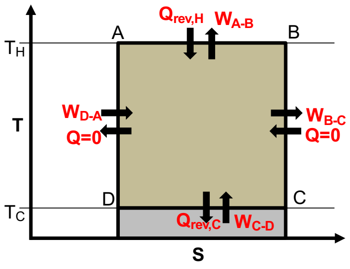
Step A-B

$$Q_{AB} = -W_{AB} =$$
$$RT_H \ln \frac{P_A}{P_B}$$

Step B-C

$$W_{BC} =$$
$$C_V (T_C - T_H)$$

The Carnot cycle



Step C-D: Isothermal compression

T= cst U=f(T)≠f(P,V) for ideal gases:

$$\Delta U = Q + W = 0 \rightarrow Q_{CD} = -W_{CD}$$
$$= \int p\, dV = RT_C \int \frac{dV}{V} =$$
$$RT_C \ln \frac{V_D}{V_C} = RT_C \ln \frac{P_C}{P_D}$$

Step A-B

$$Q_{AB} = -W_{AB} =$$
$$RT_H \ln \frac{P_A}{P_B}$$

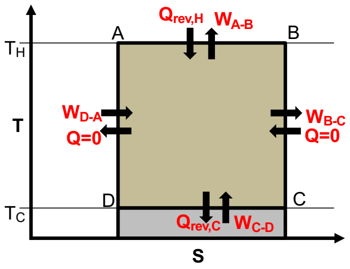
Step B-C

$$W_{BC} =$$
$$C_V (T_C - T_H)$$

Step C-D

$$Q_{CD} = -W_{CD} =$$
$$RT_C \ln \frac{P_C}{P_D}$$

The Carnot cycle



Step D-A: Adiabatic compression

U=f(T) and Cv=cst (for ideal gases):

$$\Delta U = Q + W = W_{DA} = C_V \Delta T = C_V (T_H - T_C)$$

Step A-B

$$Q_{AB} = -W_{AB} = RT_H \ln \frac{P_A}{P_B}$$

Step B-C

$$W_{BC} = C_V (T_C - T_H)$$

Step C-D

$$Q_{CD} = -W_{CD} = RT_C \ln \frac{P_C}{P_D}$$

Step D-A

$$W_{DA} = C_V (T_H - T_C)$$

The Carnot cycle

Step A-B

$$Q_{AB} = -W_{AB} = RT_H \ln \frac{P_A}{P_B}$$

Step B-C

$$\cancel{W_{BC} = C_V (T_C - T_H)}$$

Step C-D

$$Q_{CD} = -W_{CD} = RT_C \ln \frac{P_C}{P_D}$$

Step D-A

$$\cancel{W_{DA} = C_V (T_H - T_C)}$$

$$\eta = \frac{-W_{tot}}{Q_H} = \frac{-(W_{AB} + W_{BC} + W_{CD} + W_{DA})}{Q_{AB}} = \frac{-W_{AB} - W_{CD}}{Q_{AB}}$$

$$\eta = \frac{RT_H \ln \frac{P_A}{P_B} + RT_C \ln \frac{P_C}{P_D}}{RT_H \ln \frac{P_A}{P_B}}$$

The Carnot cycle

$$\eta = \frac{-W_{tot}}{Q_H} = \frac{RT_H \ln \frac{P_A}{P_B} + RT_C \ln \frac{P_C}{P_D}}{RT_H \ln \frac{P_A}{P_B}} = \frac{T_H \ln \frac{P_A}{P_B} + T_C \ln \frac{P_C}{P_D}}{T_H \ln \frac{P_A}{P_B}}$$

For ideal gases we saw that for an adiabatic step:

$$\left(\frac{P_{final}}{P_{initial}}\right)^{\frac{k-1}{k}} = \left(\frac{T_{final}}{T_{initial}}\right)$$

In our case for both adiabatic steps (A-D and B-C), we have the same interval T_H and T_C:

$$\left(\frac{P_A}{P_D}\right)^{\frac{k-1}{k}} = \left(\frac{T_H}{T_C}\right) = \left(\frac{P_B}{P_C}\right)^{\frac{k-1}{k}}$$

$$\Rightarrow \frac{P_A}{P_D} = \frac{P_B}{P_C} \Rightarrow \frac{P_A}{P_B} = \frac{P_D}{P_C}$$

The Carnot cycle

$$\eta = \frac{-W_{tot}}{Q_H} = \frac{RT_H \ln \frac{P_A}{P_B} + RT_C \ln \frac{P_C}{P_D}}{RT_H \ln \frac{P_A}{P_B}} = \frac{T_H \ln \frac{P_A}{P_B} + T_C \ln \frac{P_B}{P_A}}{T_H \ln \frac{P_A}{P_B}}$$

This results in:

$$\eta = \frac{T_H - T_C}{T_H}$$

The theoretical efficiency of heat to work engines is not 100% unless you can reach infinite temperatures or 0 K!

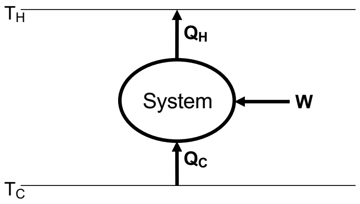
The Carnot heat pump

Sometimes it is useful to use work to move heat from a cold to a hot source

(for the reverse, you don't need work, it happens on its own)

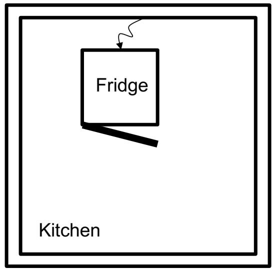
Such systems include:

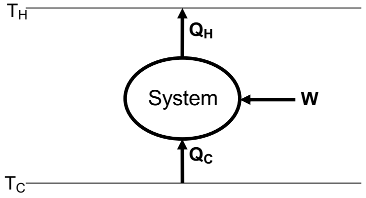
- Refrigerators
- Heat pumps
- Air conditioners



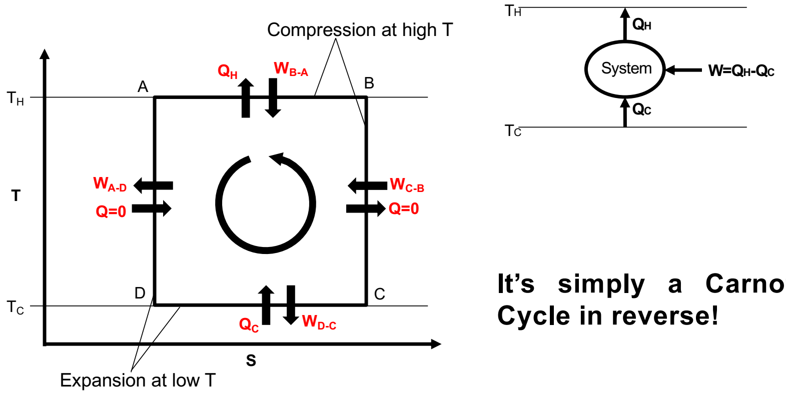
The refrigerator question

What happens if you leave for the weekend, close the door of your perfectly insulated kitchen but inadvertently leave the refrigerator door open? Does the temperature of the kitchen increase or decrease?





The Carnot heat pump



It's simply a Carnot Cycle in reverse!

The Carnot heat pump

Step B-A

$$Q_{BA} = -W_{BA} = RT_H \ln \frac{P_B}{P_A}$$

Step A-D

$$\cancel{W_{AD} = C_V(T_C - T_H)}$$

Step D-C

$$Q_{DC} = -W_{DC} = RT_C \ln \frac{P_D}{P_C}$$

Step C-B

$$\cancel{W_{CB} = C_V(T_H - T_C)}$$

In the winter:

$$COP_W = \frac{-Q_H}{W_{tot}} = \frac{-Q_{BA}}{W_{BA} + W_{DC}} = \frac{-RT_H \ln \frac{P_B}{P_A}}{-RT_H \ln \frac{P_B}{P_A} - RT_C \ln \frac{P_D}{P_C}} = \frac{T_H}{T_H - T_C}$$

The Carnot heat pump

Step B-A

$$Q_{BA} = -W_{BA} = RT_H \ln\frac{P_B}{P_A}$$

Step A-D

$$\cancel{W_{AD} = C_V(T_C - T_H)}$$

Step D-C

$$Q_{DC} = -W_{DC} = RT_C \ln\frac{P_D}{P_C}$$

Step C-B

$$\cancel{W_{CB} = C_V(T_H - T_C)}$$

In the summer:

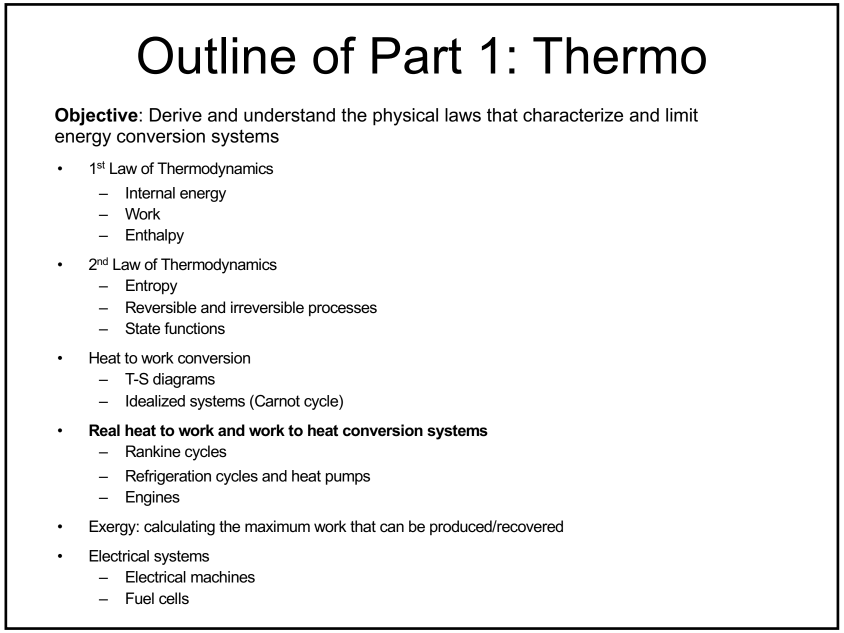
$$COP_S = \frac{Q_C}{W_{tot}} = \frac{Q_{DC}}{W_{BA} + W_{DC}} = \frac{RT_C \ln\frac{P_D}{P_C}}{-RT_H \ln\frac{P_B}{P_A} - RT_C \ln\frac{P_D}{P_C}} = \frac{T_C}{T_H - T_C}$$

ChE-304 Problem Set 2

Week 2

Problem 1

Should I heat my house with a Carnot heat pump or an electric heater? Electric heaters are very efficient. ~100% of the electrical work ends up as heat in the house.
I would like to heat my house to 24°C and the temperature outside in the winter is, on average -4°C.



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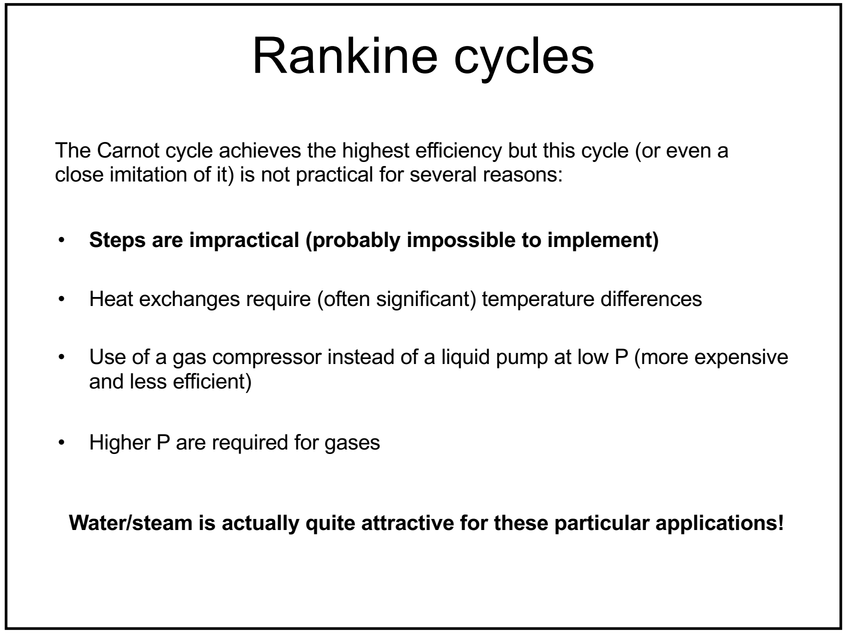


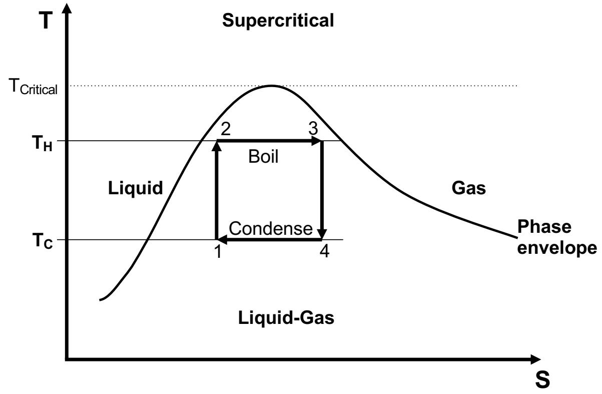
Rankine cycles

The Carnot cycle achieves the highest efficiency but this cycle (or even a close imitation of it) is not practical for several reasons:

- **Steps are impractical (probably impossible to implement)**
- Heat exchanges require (often significant) temperature differences
- Use of a gas compressor instead of a liquid pump at low P (more expensive and less efficient)
- Higher P are required for gases

Water/steam is actually quite attractive for these particular applications!

Rankine cycles



Rankine cycles

Problem: pumping and expansion with phase changes

- Pumping a liquid close to a gaseous state leads to cavitation
- Expanding a gas-to-liquid mixture leads to erosion
- Efficiency losses with formation of droplets
- Pumping a liquid is more efficient than a gas

Pumps and turbines (e.g. devices with moving elements) must be run outside of the 2-phase region

You also want to pump a liquid rather than a gas.

Rankine cycles

Cavitation damage

Turbine blades

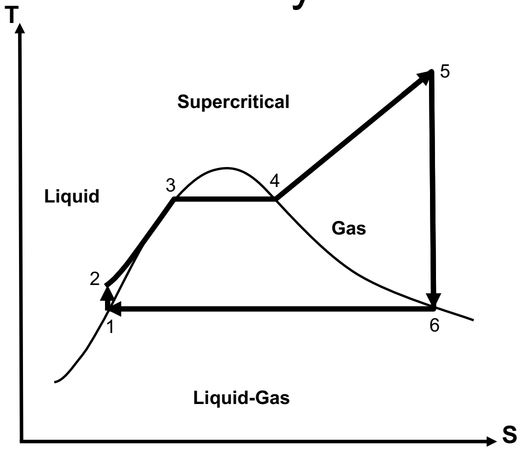
Pump plate

Droplet corrosion damage

Turbine blades

Source: Wikicommons media, http://500daysoficeland.com

Rankine cycles



Step 1-2
Liquid pumping:
Adiabatic compression

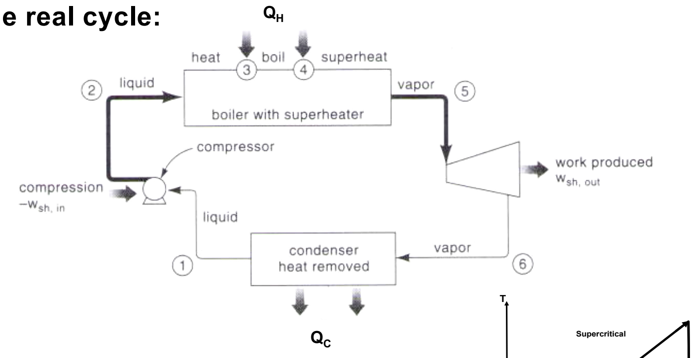
Step 2-3, 3-4, 4-5
Heating of a:
2-3: liquid
3-4: gas-liquid
4-5: gas

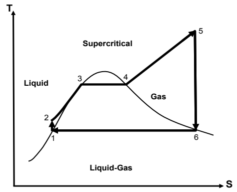
Step 5-6
Gas expansion:
Adiabatic expansion

Step 6-1
Cooling of a gas/liquid

Rankine cycles

The real cycle:



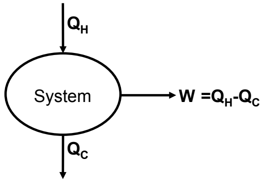


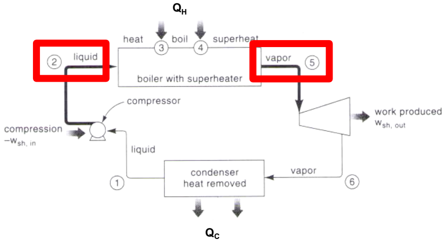
The basic principle: pressurize at low temperature, expand at high temperature → Work

Rankine cycles

Calculating efficiencies:

This view is still accurate:





The efficiency is still:

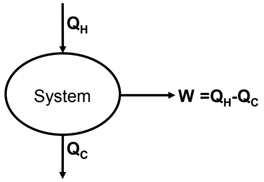
With enthalpy:

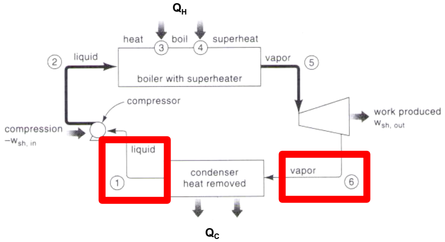
$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{\boxed{(H_5 - H_2)} - (H_6 - H_1)}{(H_5 - H_2)}$$

Rankine cycles

Calculating efficiencies:

This view is still accurate:





The efficiency is still:

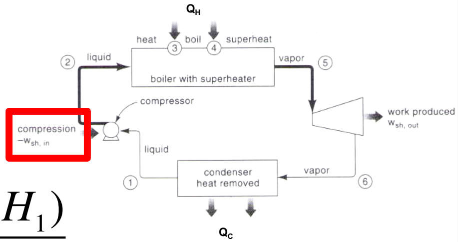
With enthalpy:

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{(H_5 - H_2) - \boxed{(H_6 - H_1)}}{(H_5 - H_2)}$$

Rankine cycles

Calculating efficiencies:

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$



$$= \frac{(H_5 - H_2) - (H_6 - H_1)}{(H_5 - H_2)}$$

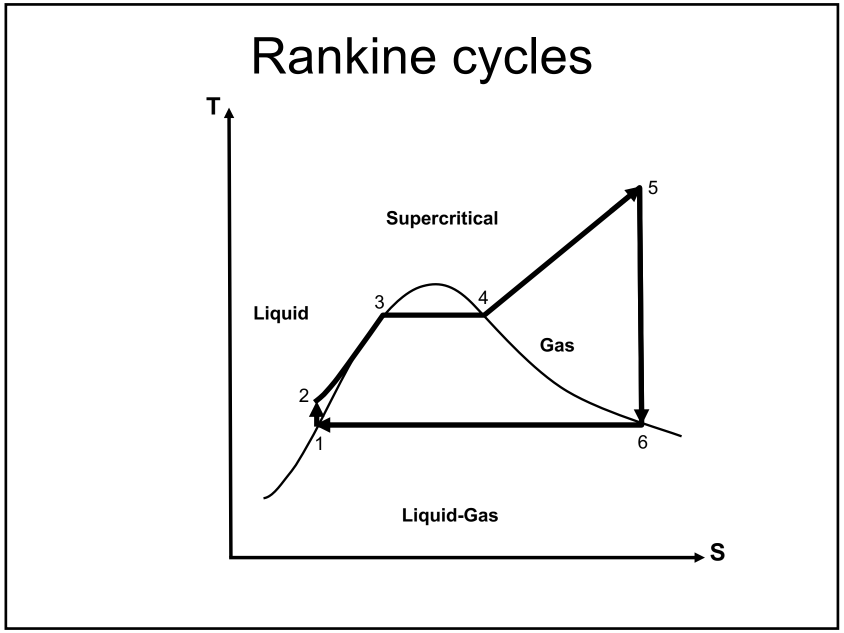
Because the energy for compressing a liquid is small:

$$H_1 \approx H_2$$

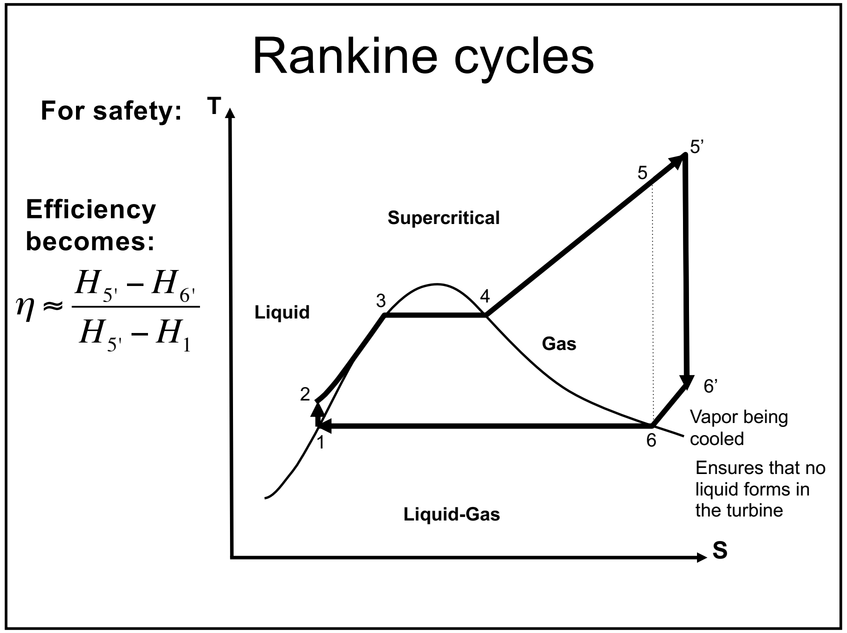
$$\longrightarrow \eta \approx \frac{H_5 - H_6}{H_5 - H_1}$$

The enthalpy change during expansion

The enthalpy change during heating



Rankine cycles



Rankine cycles

For safety:

Efficiency becomes:

$$\eta \approx \frac{H_{5'} - H_{6'}}{H_{5'} - H_1}$$

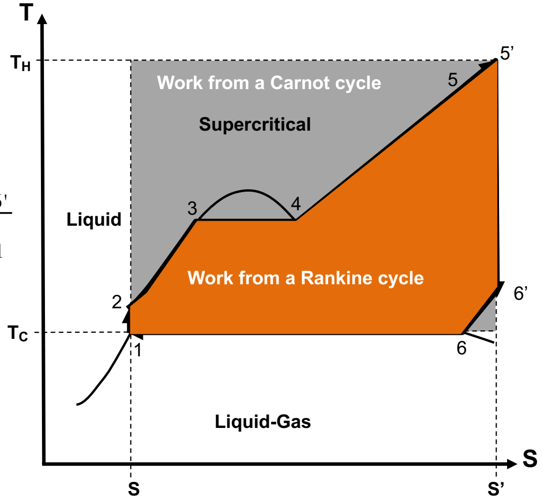
Vapor being cooled

Ensures that no liquid forms in the turbine

Rankine cycles

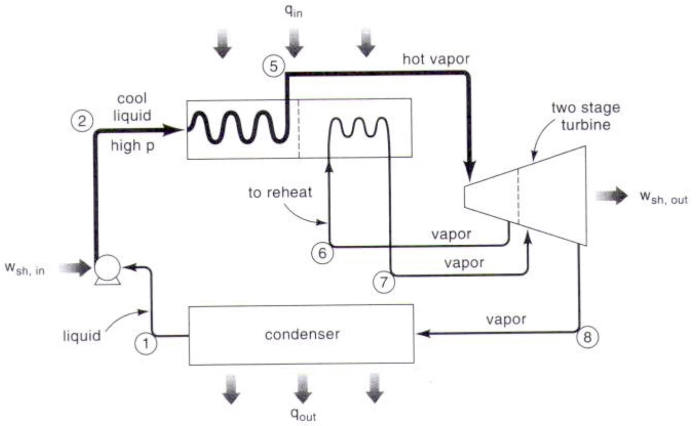
Efficiency becomes:

$$\eta \approx \frac{H_{5'} - H_{6'}}{H_{5'} - H_1}$$



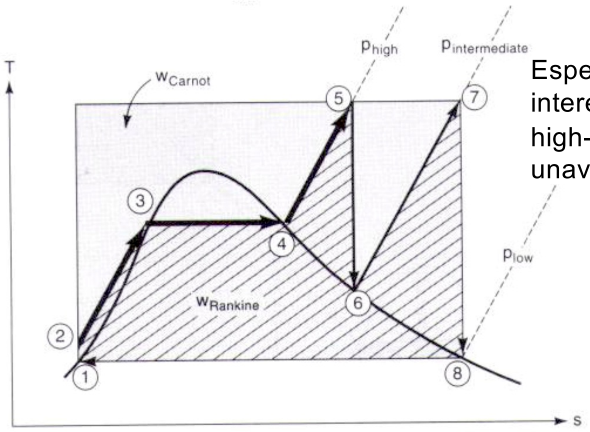
Rankine cycles

Approaching Carnot efficiency by reheating steam:



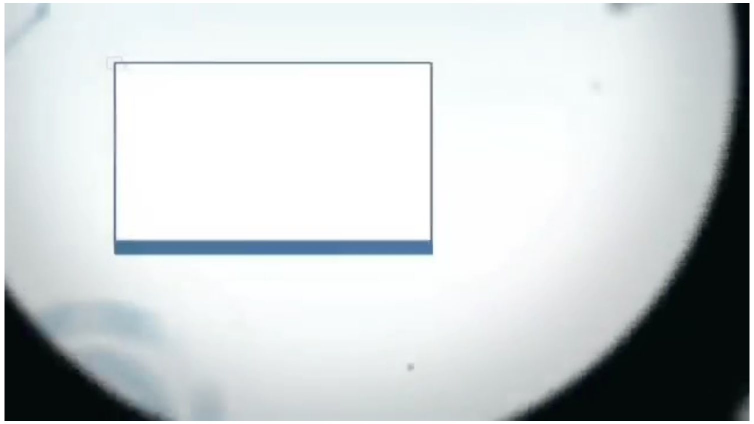
Rankine cycles

Approaching Carnot efficiency by reheating steam:

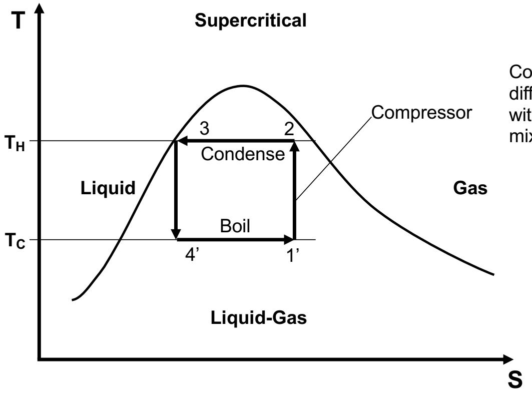


Especially interesting when high-T° source is unavailable!

Steam turbines

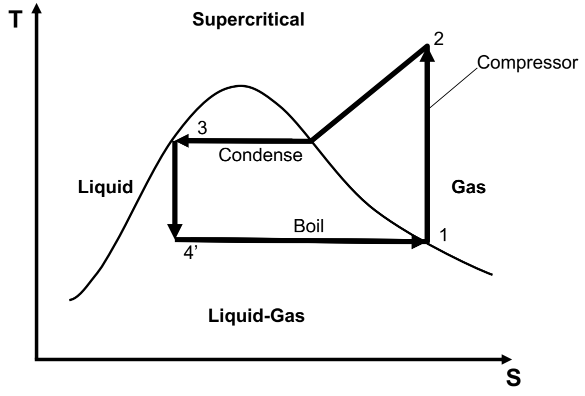


The Rankine refrigeration cycle



Compressors are difficult to run with a gas-liquid mixture!

The Rankine refrigeration cycle



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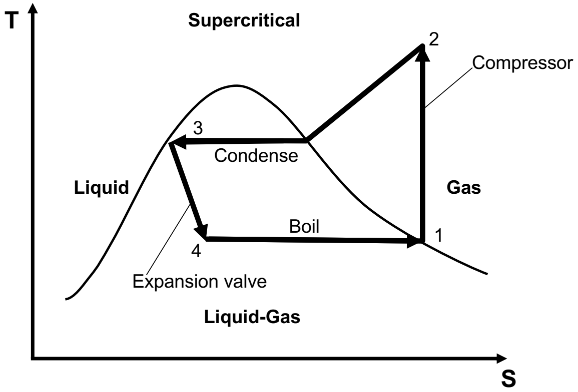
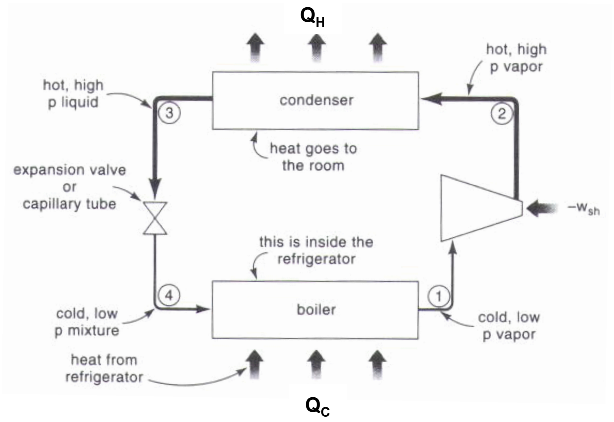
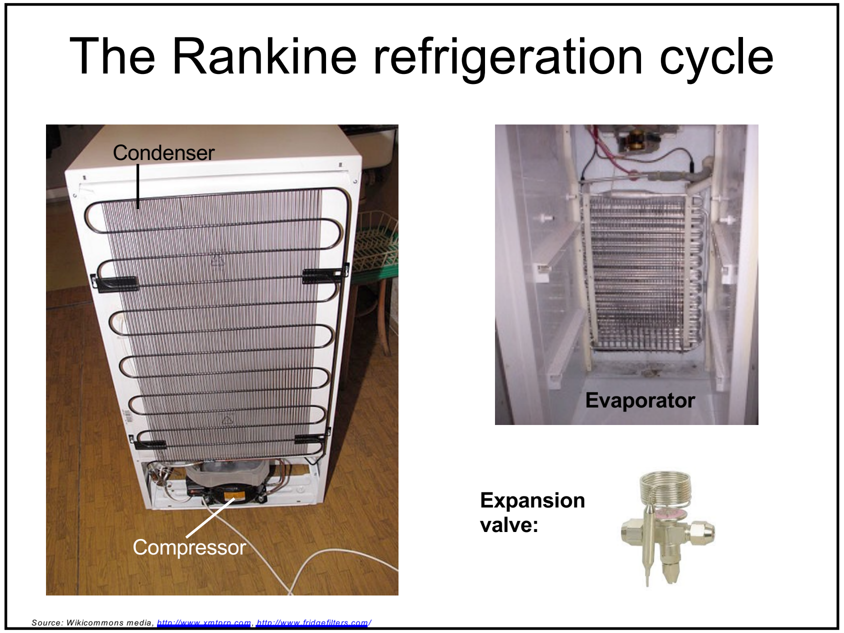


With an expansion valve the process is by definition irreversible and leads to a change in entropy!

The Rankine refrigeration cycle

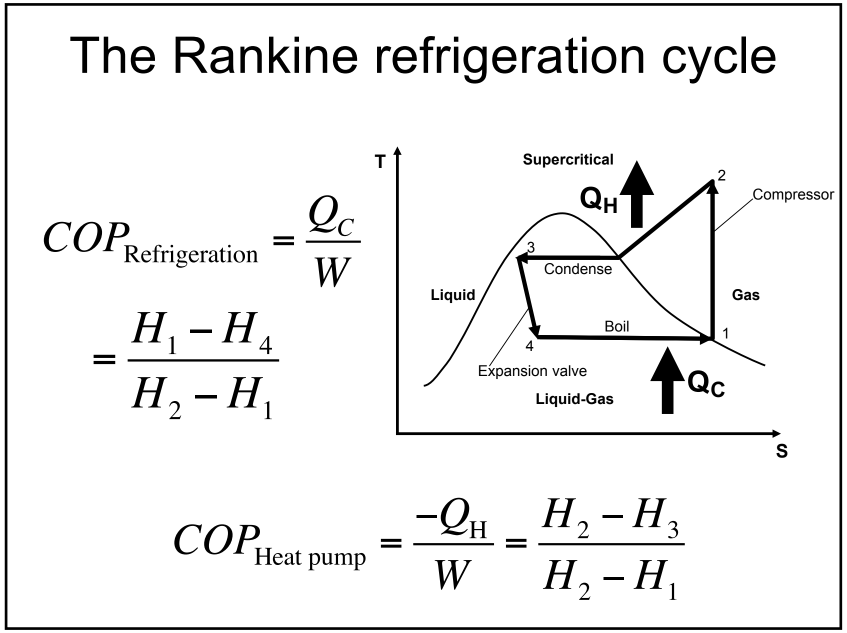




The Rankine refrigeration cycle

Expansion valve:

Source: Wikicommons media, http://www.xmtprp.com, http://www.fridgefilters.com/

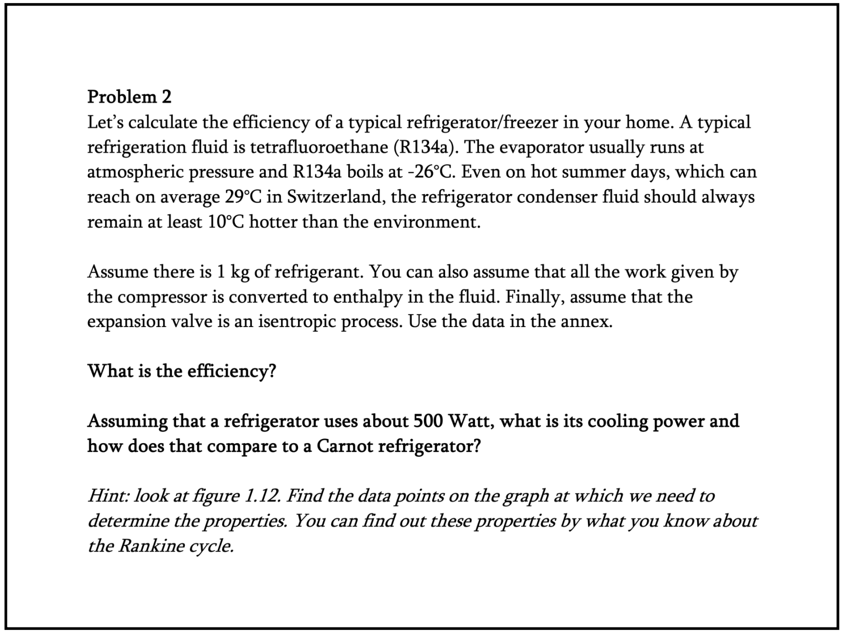


The Rankine refrigeration cycle

$$COP_{\text{Refrigeration}} = \frac{Q_C}{W}$$

$$= \frac{H_1 - H_4}{H_2 - H_1}$$

$$COP_{\text{Heat pump}} = \frac{-Q_{\text{H}}}{W} = \frac{H_2 - H_3}{H_2 - H_1}$$



Problem 2
Let's calculate the efficiency of a typical refrigerator/freezer in your home. A typical refrigeration fluid is tetrafluoroethane (R134a). The evaporator usually runs at atmospheric pressure and R134a boils at -26°C. Even on hot summer days, which can reach on average 29°C in Switzerland, the refrigerator condenser fluid should always remain at least 10°C hotter than the environment.

Assume there is 1 kg of refrigerant. You can also assume that all the work given by the compressor is converted to enthalpy in the fluid. Finally, assume that the expansion valve is an isentropic process. Use the data in the annex.

What is the efficiency?

Assuming that a refrigerator uses about 500 Watt, what is its cooling power and how does that compare to a Carnot refrigerator?

Hint: look at figure 1.12. Find the data points on the graph at which we need to determine the properties. You can find out these properties by what you know about the Rankine cycle.